# NCTM 2015 Annual Meeting & Exposition

# 712- Problems Worth Solving: Mathematical Modeling Contests and Education

Saturday, April 18, 2015: 11:00 AM-12:00 PM Lead Speaker: Solomon Garfunkel

A brief history of the HiMCM competition will be given, including a selection of problems and solutions. Attention will be paid on how to introduce modeling effectively into the high school curriculum and meet the CCSSM practices.

Presentation Format: 9-12 Session Grade Band Audience: 9 to 12



COMAP was founded in 1980 with the expressed purpose of providing materials to teach mathematics through contemporary applications and modeling.

It is an article of faith with us that for many students seeing the important practical uses of mathematics will significantly enhance their learning. And moreover, that mathematical modeling is a life skill that will serve them not only in school, but as citizens, in the work place and in their daily lives.

Over these past 35 years COMAP has produced modules, textbooks, video series, software at all educational levels which exemplify this approach and these beliefs. In some cases, such as the text For All Practical Purposes (about to go into its 10th edition) we have been extremely successful. In others we have not achieved widespread usage.

But in all cases we have provided a storehouse of material that provides educators with answers to "What is this stuff good for?"

Having exemplary curriculum materials is part of the answer for how to change educational practice. You really can't beat something with nothing. No matter how eloquently one describes a utopian curriculum with sound philosophical argument, there is nothing like handing textual materials to someone and saying, 'like this'. But changing educational practice is like moving tectonic plates and it sometimes feels like we are working in geologic time. Progress requires action on a number of fronts.

COMAP was always conceived as a grassroots bottom up organization, gathering together people of like mind, forming a core of believers and then spreading the word. But it is important to be open to a wide variety of approaches. And so when Ben Fusaro came to me in 1983 suggesting we launch an undergraduate modeling competition, we quickly moved forward with the idea.

It must be said that we have never seen modeling as a competitive sport. In fact in our original proposal to FIPSE we wrote that the rationale for the MCM was solely to "increase the presence of applied mathematics and modeling on our nation's campuses." And it has worked in ways we never imagined.

While the major portion of this talk will be about modeling activities and contests at the high school level, I want to say a little more about MCM for the historical record. MCM began in 1984 with 90 U.S. teams from 70 colleges. It is a



In 1999 we added an interdisciplinary problem







As you will note there is a distinctly higher participation of women in MCM than in more straight mathematical exams such as the Putnam.

Of course, the dramatic rise in students and teams is due to the entrance in a non-trivial way of the Chinese. In fact, in China, success on MCM is a guarantor of entrance into graduate school and/or career placement. I also honestly believe that we have, as we

promised the Department of Education, been responsible through MCM for increasing the quantity and quality of undergraduate modeling courses.

But this really is far from sufficient. As important as modeling is at the college level, by the time most students arrive in college they have lost interest in mathematics. In part we believe this is because they have not seen the breadth and relevance of what they can do with mathematics. In other words, they have not been exposed to modeling.

We very much hope that the inclusion of modeling as a mathematical practice in the Common Core and the inclusion of modeling assessment items on new high stakes tests will change that. Because as Henry Pollak has so eloquently written:

> "Mathematicians are in the habit of dividing the world into two parts: mathematics and everything else, sometimes called the 'real world'. People often tend to see these two as independent of one another – nothing could be further from the truth."

The success of MCM led us directly to seek funding from the NSF to begin a high school contest. HiMCM began in 1999. High school presents several interesting wrinkles that don't affect colleges. The first of these is the high school calendar. Calendars differ wildly from state to state, so that picking a fixed set of dates almost guarantees that some schools will not be able to compete. Secondly it's unrealistic to think students can get three or four consecutive days to work on a project. As a consequence the HiMCM rules are:



Here are the charts for team enrollment and gender over the years.







			State	Tean	ıs		
			CA	45	From	10	Schools
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		<u> </u>	CT	3	From	1	School
USA	360		FI	11	From	4	Schools
China	242		GA	16	From	3	Schools
China	313		HI	3	From	1	School
Finland	2		IA	76	From	14	Schools
i mana	-		IL	56	From	8	Schools
Hona Kona	25		MA	4	From	5	Schools
			MD	3	From	1	School
Jordan	1		MI	7	From	2	Schools
C:	2		MS	9	From	2	Schools
Singapore	3		MT	11	From	2	Schools
Turkov	1		NT	2	From	4	Schools
Turkey	-		NC	12	From	3	Schools
Total	705		NH	6	From	2	Schools
			он	2	From	1	School
			PA	7	From	4	Schools
				32	From	1	Schoole
			wi	6	From	2	Schools
				360	From	79	Schools
				360	From	24	States

You will note that foreign (Chinese) teams are just now beginning to increase. But for HiMCM as opposed to MCM we may not immediately see the exponential growth of participation. This is because in China as in many other foreign countries there is an end of high school test that is truly high stakes, i.e. a test which can determine a student's

entire future. And these tests tend to be very conservative when it comes to applications and modeling. It is difficult therefore to introduce secondary school modeling materials when they are not included on this crucial assessment. More on this later.

But we have spent a lot of time talking about modeling contests and problems without seeing what they look like. Here are just a few from past HiMCM contests:

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#### Jan. 2001 Problem A

#### Design of an Airline Terminal

The design of airline terminals varies widely. The sketches below show airline terminals from several cities. The designs are quite dissimilar. Some involve circular arcs; others are rectangular; some are quite irregular. Which is optimal for operations? Develop a mathematical model for airport design and operation. Use your model to argue for the optimality of your specified design. Explain how it would operate.



Pittsburgh International

## Nov. 2001 Problem B

## Skyscrapers

Skyscrapers vary in height, size (square footage), occupancy rates, and usage. They adorn the skyline of our major cities. But as we have seen several times in history, the height of the building might preclude escape during a catastrophe either human or natural (earthquake, tornado, hurricane, etc). Let's consider the following scenario. A building (a skyscraper) needs to be evacuated. Power has been lost so the elevator banks are inoperative except for use by firefighters and rescue personnel with special keys.

Build a mathematical model to clear the building within X minutes. Use this mathematical model to state the height of the building, maximum occupation, and type of evacuation methods used. Solve your model for X = 15 minutes, 30 minutes, and 60 minutes.

#### 2004 Problem A

#### Motel Cleaning Problem

Motels and hotels hire people to clean the rooms after each evening's use. Develop a mathematical model for the cleaning schedule and use of cleaning resources. Your model should include consideration of such things as stay-overs, costs, number of rooms, number of rooms per floor, etc. Draft a letter to the manger of a major motel or hotel complex that recommends your model to help them in the management of their operation.

# 2007 Problem B

# **Car Rentals**

Some people rent a car when they are going on a long trip. They are convinced they save money. Even if they do not save money, they feel that the knowledge that "if the car breaks down on the trip, the problem is the rental company's" makes the rental worth it. Analyze this situation and determine under what conditions renting a car is a more appropriate option. Determine mileage limits on one's own car and a break-even value of "ease of mind" for the driver and her family.

Outstanding solution papers are available and samples are published in our Consortium newsletter each year.

Here is the flyer for this year's upcoming contest:



With the success of these contests and the introduction of modeling into the CCSSM there are still two important questions to answer. How do we promote the teaching and learning of mathematical modeling in an authentic way and what does the teaching of modeling look like day to day, grade by grade in a practical way.

COMAP is attempting to address both of these issues with two new initiatives - **GAIMME** and **IMMC**.

## **GAIMME – Guidelines for Assessment and Instruction in Mathematical Modeling Education**

A joint project of COMAP and SIAM

Taking off from the excellent GAISE report of the ASA, this report will have three main sections:



This report will be out no later than the end of the year, hopefully by the end of the summer and there will be a major address and/or panel devoted to it at next year's annual meeting.

The project teams are being led by:

Rachel Levy (elementary) of Harvey Mudd College Dan Teague (high school) of NCSSM Frank Giordano (undergraduate) of the Naval Postgraduate School

With a distinguished and experienced group of 15 writers and some 20 reviewers.

This report is being written for classroom teachers – to be of direct use in creating honest modeling tasks. And it is in particular being written for those creating high stakes assessments, so that we are assured to test what we value. Modeling is not the same as word problems, nor is it the same as applications of specific techniques. It is a life skill, a process for making sense of the world through the use of mathematical tools and methodology.

The second new major initiative I want to announce is the IMMC – The International Mathematics Modeling Challenge.



While HiMCM is open to teams from high schools anywhere in the world and the Moody's contest will soon move to allow teams from abroad, there is nothing in modeling on a par with the Math Olympiad. And while I continue to believe that mathematics and mathematical modeling are not competitive sports, there is no question that high profile 'challenges' can have a powerful effect on what we teach. MCM is certainly proof at the college level.

It is also our hope that as IMMC grows in size and prestige it can help break the curricular grip of technique oriented high stakes tests and make more room for applications and modeling. In my opinion it is well past time for comments such as, "My curriculum is too crowded; modeling takes too much time; I have to cover the techniques that will be on the test."

IMMC is a work in progress. The first contest began on April 15, 2015. And just as with HiMCM we had to adjust to a world full of crowded and different calendars. In order to accommodate scheduling issues we developed the following rules:



This year we have 19 teams from 11 countries:



This year and next we will have only a written test, but in subsequent years we plan to have a second round where the country representative teams (up to 2 per country) will come to a host country (that will change each year) to take part in joint modeling activities. Information about the Challenge can be found at <u>www.immchallenge.org</u>

We invite you all to take part as modeling becomes an increasingly essential part of the mathematics curriculum at all levels.

